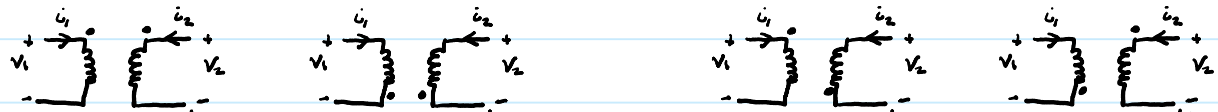
2018-09-26-1

**Last time:** Dot Convention

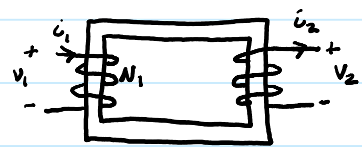
* Dot location doesn't effect self inductance term voltage drop




$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$ $\quad$ $v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$ $\quad$ $v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$ $\quad$ $v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$

$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$ $\quad$ $v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$ $\quad$ $v_2 = -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$ $\quad$ $v_2 = -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$

Ideal transformer:




$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = a$$

$$\frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{1}{a}$$

$$P_1 = P_2$$

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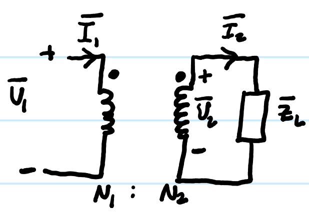
**Today:**
1) Ideal transformer and impedences
2) Max power transfer
3) Equivalent circuit for transformer

**Impendences and Transformers:**




$\bar{V}_2 = \bar{Z}_L \bar{I}_2$

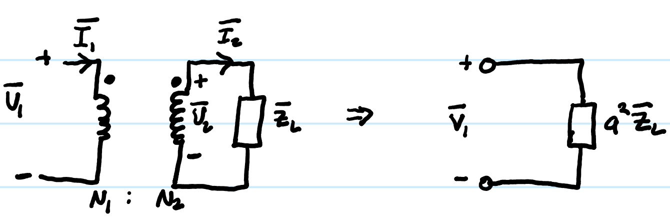
$\bar{V}_1 = a\bar{V}_2$

$\bar{V}_1 = a\bar{Z}_L \bar{I}_2$

$\bar{I}_1 = \frac{1}{a}\bar{I}_2 \Rightarrow \bar{I}_2 = a\bar{I}_1$

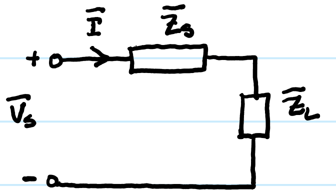
$\bar{V}_1 = a^2 \bar{Z}_L \bar{I}_1$

$$\boxed{\bar{V}_1 = (a^2 \bar{Z}_L)\bar{I}_1}$$




* Impedence on 2 side can be moved to 1 side by multiplying by $a^2$.

## Maximum power transfered to the load




$\bar{Z}_s = R_s + jX_s$

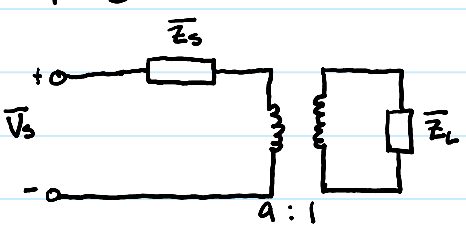
$\bar{Z}_L = R_L + jX_L$

$$P_L = |\bar{I}|^2 R_L$$

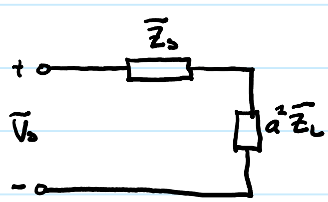
$$\bar{I} = \frac{\bar{V}_s}{\bar{Z}_s + \bar{Z}_L} \Rightarrow \bar{I} = \frac{\bar{V}_s}{(R_s+R_L)+j(X_s+X_L)} \qquad P_L = \frac{|\bar{V}_s|^2 R_L}{(R_s+R_L)^2 + (X_s+X_L)^2}$$

* Can show that for max $P_L$, $\bar{Z}_L = \bar{Z}_s^*$ or $|\bar{Z}_L| = |\bar{Z}_s|$ (for $\frac{R_L}{X_L}$ fixed as a constant)

* Imposing $|\bar{Z}_L| = |\bar{Z}_s|$ can easily be done by using a transformer



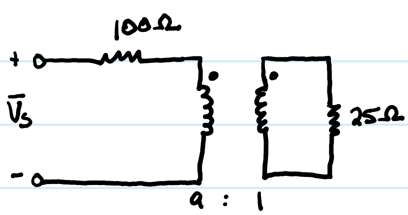

$\Rightarrow$




$$|a^2 \bar{Z}_L| = |\bar{Z}_s| \Rightarrow a^2 |\bar{Z}_L| = |\bar{Z}_s|$$

$$a^2 = \frac{|\bar{Z}_s|}{|\bar{Z}_L|} \Rightarrow \boxed{a = \sqrt{\frac{|\bar{Z}_s|}{|\bar{Z}_L|}}}$$

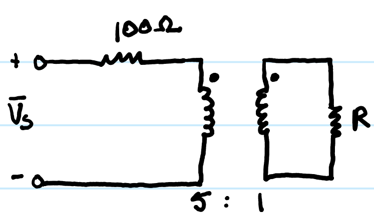
**Ex**




Find: a for max power transfer

$$a = \sqrt{\frac{100}{25}} = a = \sqrt{4} \Rightarrow \boxed{a = 2}$$

**Ex**

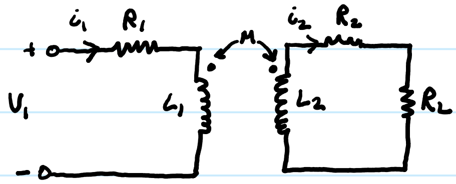



Find: R for max power transfer

$$5^2 R = 100 \Rightarrow 25R = 100\,\Omega$$

$$\boxed{R = 4\,\Omega}$$

## Equivalent circuits for transformers

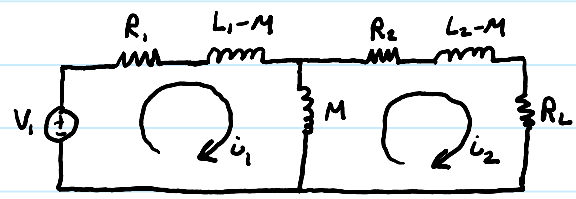



$V_1 = i_1R_1 + L_1\frac{di_1}{dt} - M\frac{di_2}{dt}$ $\Rightarrow$ $V_1 = i_1R_1 + (L_1-M)\frac{di_1}{dt} + M\frac{d}{dt}(i_1-i_2)$

$0 = i_2R_2 + i_2R_L + L_2\frac{di_2}{dt} - M\frac{di_1}{dt}$ $\Rightarrow$ $0 = i_2R_2 + (L_2-M)\frac{di_2}{dt} + M\frac{d}{dt}(i_2-i_1) + i_2R_L$

* $i_1$: dot → no dot. Coil 2: current into dot $= -i_2$

* $i_2$: no dot → dot. Coil 1: current into no dot $= -i_1$




* if $L_1-M$ or $L_2-M$ are less than 0, then the above equation shouldn't be used.

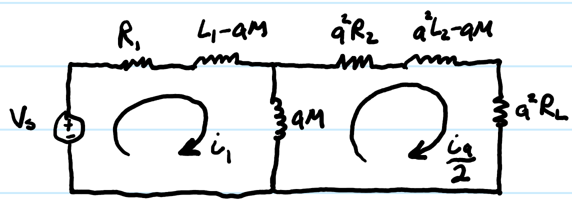
* More generally:

$V_1 = i_1R_1 + L_1\frac{di_1}{dt} - M\frac{di_2}{dt}$ $\Rightarrow$ $V_1 = i_1R_1 + L_1\frac{di_1}{dt} - aM\frac{d}{dt}\left(\frac{i_2}{a}\right)$

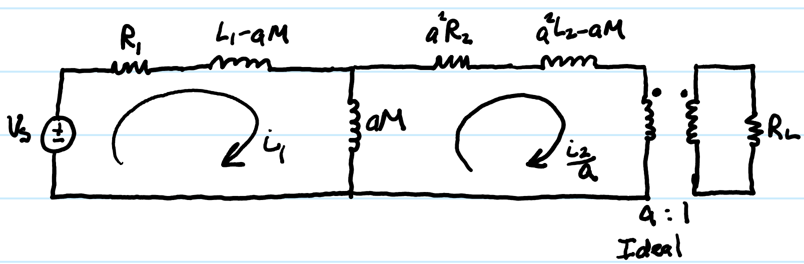
$0 = i_2R_2 + i_2R_L + L_2\frac{di_2}{dt} - M\frac{di_1}{dt}$ $\quad$ $0 = a\left(\frac{i_2}{a}\right)R_2 + a\left(\frac{i_2}{a}\right)R_L + aL_2\frac{d}{dt}\left(\frac{i_2}{a}\right) - M\frac{di_1}{dt}$

$\Rightarrow V_1 = i_1R_1 + (L_1 - aM)\frac{di_1}{dt} + aM\frac{d}{dt}\left(i_1 - \frac{i_2}{a}\right)$

$0 = a^2\left(\frac{i_2}{a}\right)R_2 + (a^2L_2 - aM)\frac{d}{dt}\left(\frac{i_2}{a}\right) + aM\left(\frac{i_2}{a} - i_1\right) + a^2\left(\frac{i_2}{a}\right)R_L$




### Thevenin Equivalent



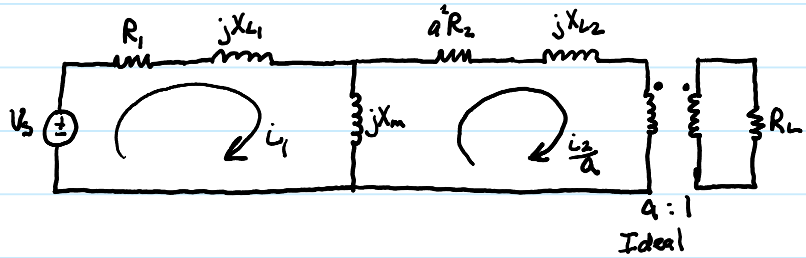
2018-09-26-4

## Analysis for sinusoidal steady state

$\bar{V}_1 = V_m \cos(\omega t + \theta_v)$

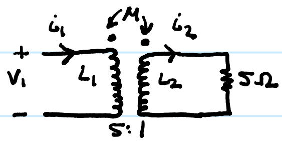
Steps: 1) Convert all inductances to reactances

$X_L = \omega L$

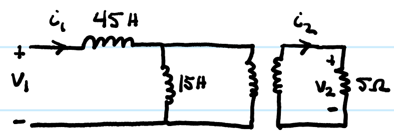



2) Solve the circuit using phasors.

## Ex| Problem 3.12




⇒




$aM = 15H \Rightarrow M = 3H$

$L_1 - aM = 45H \Rightarrow L_1 = 60H$

$a^2L_2 - aM = 0 \Rightarrow L_2 = \frac{M}{a} \Rightarrow L_2 = \frac{3}{5} = 0.6H$

$k = \frac{M}{\sqrt{L_1 L_2}} \Rightarrow k = 0.5$